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BIRZEIT UNIVERSITY

Mathematics Department

Math 234

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Test One

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Question One (14 points): Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & \alpha \\ 1 & \alpha & 3 \end{pmatrix}$. Circle the most correct answer:

1. The cofactor A_{32} of a_{32} is

   (a) $\alpha - 1$

   (b) $\boxed{1-\alpha}$

   (c) $27-\alpha^2$

   (d) $\alpha - 9$

$(a-1)$

$-a+1$

$1-a$

$-(\alpha-1)$

$-1-9$

2. The value of α that makes the matrix A singular is

   (a) 1

   (b) 2

   (c) -3 or 5

   (d) 4 or 5

$\sum_{k=1}^{3} a_{1k} A_{1k}$

$= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$1 \times \begin{vmatrix} 9 & a \\ a & 3 \end{vmatrix} + -1\begin{vmatrix} 1 & a \\ 1 & 3 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 9 \\ 1 & a \end{vmatrix}$

$(27-a^2) + 3 - a + a - 9$

$27 - a^2 + a - 3 + a - 9$

$-a^2 + 2a + 15$

$a^2 - 2a - 15$

$(a+3)(a-5)$

3. If $\alpha = 3$, then A^{-1} is

   (a) not defined

   (b) $\begin{pmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{2}{3} \end{pmatrix}$

   (c) $\begin{pmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{3}{2} \end{pmatrix}$

   (d) none of the above

4. If $\alpha = 3$ and $\vec{b} = (6,6,6)$, then

(a) the system $A\mathbf{x} = \mathbf{b}$ is inconsistent

(b) the solution set of the system $A\mathbf{x} = \mathbf{b}$ is $\{(\beta,0,0) : \beta \in \mathbb{R}\}$

(c) the system $A\mathbf{x} = \mathbf{b}$ has a unique solution $(6,0,0)$

(d) none of the above

$AX = B$
$X = A^{-1}B$

$$\begin{bmatrix} \frac{3}{2} & 0 & \frac{-1}{2} \\ 0 & \frac{1}{6} & \frac{-1}{6} \\ \frac{-1}{2} & \frac{-1}{6} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

5. If $\alpha = 3$, then adj A is

(a) $\begin{pmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{2} & -\frac{1}{6} & \frac{2}{3} \end{pmatrix}$

(b) $\begin{pmatrix} 18 & 0 & -6 \\ 0 & 2 & -2 \\ -6 & -2 & 8 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & 3 \\ 1 & 3 & 3 \end{pmatrix}$

(d) none of the above

6. Let $\alpha = 3$. The LU factorization of the matrix $\cancel{B}$ A implies that $L = E_1^{-1}E_2^{-1}E_3^{-1}$, where

(a) $L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -\frac{1}{4} & 1 \end{pmatrix}$

(b) $\boxed{E_1^{-1}} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(c) $E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

(d) $E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{4} & 1 \end{pmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 9 & 3 \\ 1 & 3 & 3 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \end{matrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 9 & 3 \\ 1 & 3 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 8 & 2 \\ 0 & 2 & 2 \end{bmatrix} R_3 - \frac{2}{8}R_2$$

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 8 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & \frac{2}{8} & 1 \end{bmatrix}$$

7. The matrix A is

(a) skew symmetric

(b) $\boxed{\text{symmetric}}$

(c) triangular

(d) none of the above

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Question Two (4 points): Use Gauss-Jordan reduction to solve the following system

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 - x_2 - 2x_3 &= 1 \\ 2x_1 + 3x_2 - 5x_3 &= -3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & -3 \end{array}\right] \begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & -5 \\ 0 & -1 & 1 & 1 \end{array}\right] R_3 - \frac{1}{7}R_2$$

$1 - 1 = 0$

$-1 - (-1)$

$-1 + 1 = 0$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & -5 \\ 0 & 0 & 0 & 0 \end{array}\right] * \frac{-1}{7}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & \boxed{-1} & \frac{5}{7} \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 - 3R_2 \\ \text{pivot} \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & \frac{-29}{7} \\ 0 & 1 & -1 & \frac{5}{7} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{2}{1} - \frac{15}{7}$

$\cdot \frac{29}{7}$

x_3 free variable

$x_3 = \alpha$ 1/2

$X_2 - X_3 = \frac{5}{7}$

$X_2 - \alpha = \frac{5}{7}$

$\boxed{X_2 = \frac{5}{7} + \alpha}$ 1/2 $\alpha - 1$

$X_1 - X_2 = \frac{-29}{7}$

$X_1 - \frac{5}{7} - \alpha = \frac{-29}{7}$

$X_1 = \frac{-29}{7} + \frac{5}{7} + \alpha$

$X_1 = \frac{-24}{7} + \alpha$ 1/2

the Solution set is $\left\{\left(\frac{-24}{7} + \alpha, \frac{5}{7} + \alpha, \alpha\right) \alpha \in R\right\}$.

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Question Three (2 points): Let A be an $n \times n$ skew-symmetric matrix. Show that if n is odd, then A must be singular.

~~by Contradiction~~

~~A $n \times n$ skew symmetric matrix, n is odd~~

~~but A is nonsingular~~

~~skew symmetric $A^T = -A$~~ ~~both side by A^{-1}~~

~~$A^{-1} A^T = A^{-1} A$~~

Good Luck